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# A new mean field $S_A$–$S_{A'}$ critical point in a symmetry breaking field

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We consider a $S_A$–$S_{A'}$ critical point in the presence of a symmetry-breaking external magnetic (electric) field with a positive magnetic (dielectric) anisotropy or a dislocation layer. Via a renormalization group analysis of the model hamiltonian, we show that the upper critical dimensions below which mean-field theory breaks down is $d_c' = 2{\cdot}5$. Thus the $S_A$–$S_{A'}$ transition in three dimensions becomes mean-field like in the presence of a symmetry-breaking field. We estimate the reduced temperature region where we can expect to see the mean field $S_A$–$S_{A'}$ critical point in the presence of a magnetic field or a dislocation layer.

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The smectic ($S_A$) liquid crystal phase is a one-dimensional solid with a density modulation in the direction parallel to the equilibrium director $\mathbf{n}_0$ ($z$-axis). Recently, extensive theoretical [1] and experimental [2] studies of so-called 'frustrated smectic liquid crystal phases', which are composted of strongly polar molecules, have revealed a large number of $S_A$ phases. In particular, there are the linear $S_{A_1}$, $S_{A_d}$, and $S_{A_2}$ phases where the indices 1, $d$, and 2 indicate that the wavelength of the periodic modulation is one, $d(1 < d < 2)$, or two times the molecular length $l$. The associated wavenumber of the modulation is, respectively, $q_0 = 2\pi/l$, $2\pi/dl$ and $2\pi/2l$, in the three cases.

Phase transitions between these $S_A$ phases are of some interest and need further comment. There can be a first order transition in which there is a discontinuous change in $q_0$ from $q_0^+$ to $q_0^-$ (e.g. an $S_{A_1}$–$S_{A_d}$ or $S_{A_d}$–$S_{A_2}$ transition) as well as second order transitions in which the amplitude $\psi_{k_1}$ of the mass density at wavenumber $k_1 = 2\pi/2l$ grows continuously from zero. In the former case, as in the liquid-gas transition, the identical macroscopic symmetry of $S_A$ phases allows the first order transition line to terminate in a critical point, as shown in the figure, where the difference $q_0 = q_0^+ - q_0^-$ in the wavenumber goes to zero. Such a critical point provides a continuous path between coexisting $S_{A_1}$ (or $S_{A_2}$) and $S_{A_d}$ phases.

This $S_A$–$S_{A'}$ critical point, C was predicted theoretically by Barois *et al.* [3] and observed in a series of experiments by Shashidhar *et al.* [4] in the binary phase diagram of (11OPCBOB/9OBCB). Previous experiments [5] providing evidence for its possible existence were not conclusive [5]. Alternatively, the first order $S_A$–$S_{A'}$ transition line can terminate on a closed re-entrant nematic domain [6]. Prost and Toner [7] predicted

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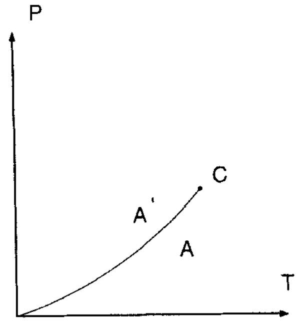


Phase diagram showing the $S_A$–$S_{A'}$ critical point C. The two smectic phases (denoted by A and A′ in the figure) coexist along a line in the concentration (or pressure)–temperature plane terminating at the critical point C.

the existence of both a 'nematic island in a smectic sea' and a $S_A$–$S_{A'}$ critical point using a dislocation loop theory of the $S_A$–N transition. Park *et al.* [8] developed a nonlinear elastic model to describe the $S_A$–$S_{A'}$ critical point and found that this critical point belonged to a new universality class with an upper critical dimension, $d_c$ of 6 (as opposed to $d_c = 4$ for the liquid-gas critical point).

In this paper, we consider the $S_A$–$S_{A'}$ transition in the presence of a symmetry breaking magnetic (electric) field with positive magnetic (dielectric) anisotropy. The Landau–Ginzburg–Wilson Hamiltonian describing this transition as a function of the displacement variable $u(\mathbf{x})$ of the smectic layers can be expressed as

$$H[u(\mathbf{x})] = H_{sm}[u(\mathbf{x})] + H_1. \tag{1}$$

Here, $H_{sm}[u(\mathbf{x})]$ is the nonlinear elastic energy of elastic deformations in smectic A phases which has the following form:

$$H(u) \equiv \int d^d x\{hE(u) + \frac{1}{2}BE^2(u) + \frac{1}{3!}wE^3(u) + \frac{1}{4!}vE^4(u) + \tfrac{1}{2}[K_1(\nabla_\perp^2 u)^2 + K_2(\nabla_z^2 u)^2 + 2K_{12}(\nabla_z\nabla_\perp u)^2]\}. \tag{2}$$

with

$$E[u(\mathbf{x})] = \nabla_z u + \tfrac{1}{2}(\nabla u)^2 \tag{3}$$

$H_1[u]$ is the symmetry breaking hamiltonian arising from an external magnetic **H** or electric field **E**. It is expressed as

$$H_1 = \int d^d x B_{20}(\nabla_\perp u)^2 \tag{4}$$

where $B_{20} = \chi_a H^2\,(\varepsilon_a E^2)$ with $\chi_a > 0\ (\varepsilon_a > 0)$. Since the independent variable is $u(\mathbf{x})$ not $E[u(\mathbf{x})]$, it is convenient for future analysis to re-express $H[u(\mathbf{x})]$ in terms of $\nabla_z u$ and $\nabla_\perp u$

$$\begin{aligned} H = \int d^d x\{& h(\nabla_z u) + \tfrac{1}{2}B_1(\nabla_z u)^2 + \tfrac{1}{2}B_2(\nabla_\perp u)^2 \\ &+ \tfrac{1}{2}[K_1(\nabla_\perp^2 u)^2 + K_2(\nabla_z^2 u)^2 + 2K_{12}(\nabla_z\nabla_\perp u)^2] \\ &+ \frac{1}{3!}w_1(\nabla_z u)^3 + \frac{1}{2}w_2(\nabla_z u)(\nabla_\perp u)^2 + \frac{1}{4!}v_1(\nabla_z u)^4 \\ &+ v_2\frac{1}{4!}(\nabla_\perp u)^4 + \frac{1}{12}v_{12}(\nabla_z u)^2(\nabla_\perp u)^2\}, \end{aligned} \tag{5}$$

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where

$$\left.\begin{aligned} B_1 &= B + h, & w_1 &= w + 3B, & v_1 &= v + 6w + 3B,\\ B_2 &= h + B_{20}, & w_2 &= B, & v_2 &= 3B,\\ & & & & v_{12} &= 3w + 3B. \end{aligned}\right\} \tag{6}$$

This is the hamiltonian we will use in the remainder of this paper. (Note that $B_2 \neq 0$ even in the absence of the symmetry breaking external field ($h = 0$).)

As discussed in [8] the order parameter for the $S_A$–$S_{A'}$ transition is $M_z = \langle \nabla_z u \rangle$. If $\nabla_\perp u = 0$ in equation (5), the resulting hamiltonian in terms of $\nabla_z u$ alone is identical in form to that describing the liquid–gas transition as a function of its scalar order parameter $\phi = n_l - n_g$, the difference in densities of the liquid and gas phases. Thus, in mean-field theory, the $S_A$–$S_{A'}$ transition occurs at $h = B_1 = w_1 = 0$ and is identical to the liquid gas transition.

Mean field theory is valid above an upper critical dimension $d_c$ below which fluctuations become important. In the LGW hamiltonian describing the liquid gas transition, there is a single third order potential which can be removed by shifting the order parameter. The upper critical dimension $d_c = 4$ is, therefore, the dimension at which the fourth order potential becomes relevant. There are two third order potentials in the hamiltonian (equation (5)) describing the $S_A$–$S_{A'}$ transition. In the absence of an external symmetry breaking field ($B_{20} = 0$), the mean field propagator is proportional to $\nabla^{-4}$ at the critical point (since $h = B_1 = 0$). The $S_A$–$S_{A'}$ critical point is thus described by renormalization group transformations that leave the coefficients of $(\nabla_z^2 u)^2$, $(\nabla_\perp^2 u)^2$ and $(\nabla_z^2 u)(\nabla_\perp^2 u)$ invariant. For $d > d_c$, $u(\mathbf{q})$, the Fourier transform of $u(\mathbf{x})$, then transforms as $u(\mathbf{q}) \to b^{d-4} u(b\mathbf{q})$, and the potentials $w_1$ and $w_2$ transforms as $w_1' = b^{-(d-6)/2} w_1$ and $w_2' = b^{-(d-6)/2} w_2$. Thus both $w_1$ and $w_2$ become relevant for $d < 6$. At the liquid gas transition, there is a single third order potential that can be removed by shifting the order parameter [9]. It is impossible to remove both potentials $w_1$ and $w_2$ at the $S_A$–$S_{A'}$ critical point via a shift of the order parameter. The upper critical dimension for the $S_A$–$S_{A'}$ transition in the absence of external fields is, therefore, 6 and not 4 as the analogy with the liquid gas transition would indicate. The properties of this very complex critical point were analysed in an $\varepsilon$-expansion about six dimensions in [8].

In the presence of external fields, $B_{20}$ is non-zero, and the rescaling of lengths parallel and perpendicular to the smectic layers is different even for $d > d_c$. To find $d_c$, we will now determine the renormalization group recursion relations to zero loop order for $B_2 \neq 0$. At the critical point, $B_1 = 0$, and we rescale to keep $B_{20}$ and $K_1$ constant. Under the transformations

$$q_\parallel \to \exp\left(-(1 + \mu_\parallel)l\right) q_\parallel, \quad q_\perp \to \exp(-l) q_\perp,$$
$$u(\mathbf{q}) \to \exp[(d + 2 + \mu_\parallel - \eta_\perp)l/2] u(b\mathbf{q}), \tag{7}$$

we have

$$\left.\begin{aligned} \frac{dB_2}{dl} &= -\eta_\perp B_2,\\ \frac{dK_1}{dl} &= (-2 - 4\mu_\parallel - \eta_\perp) K_1. \end{aligned}\right\} \tag{8}$$

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When $d < d_c$, $B_2$ and $K_1$ should be constant under the above transformations. This requires $\mu_\parallel = -\frac{1}{2} + \mu'_\parallel$ where $\mu'_\parallel$ and $\eta_\perp$ are zero for $d > d_c$ and of order $\varepsilon = d_c - d$ for $d < d_c$. With this choice for $\mu'_\parallel$ we find to zero loop order

$$\left.\begin{aligned}
\frac{dB_1}{dl} &= (1 - 2\mu'_\parallel - \eta_\perp)B_1 + \cdots, \\
\frac{dB_2}{dl} &= -\eta_\perp B_2 + \cdots, \\
\frac{dK_1}{dl} &= (-4\mu'_\parallel - \eta_\perp)K_1, \\
\frac{dw_1}{dl} &= [\tfrac{1}{2}(\tfrac{7}{2} - d) - \tfrac{7}{2}\mu'_\parallel - \tfrac{3}{2}\eta_\perp]w_1 + \cdots, \\
\frac{dw_2}{dl} &= [-1 + \tfrac{1}{2}(\tfrac{7}{2} - d) - \tfrac{3}{2}\mu'_\parallel - \tfrac{3}{2}\eta_\perp]w_2 + \cdots, \\
\frac{dv_1}{dl} &= [(-d + \tfrac{5}{2}) - 5\mu'_\parallel - 2\eta_\perp]v_1 + \cdots.
\end{aligned}\right\} \tag{9}$$

These equations are sufficient to determine the upper critical dimension. One loop corrections are needed to determine exponents to first order in $d_c - d$. All other potentials (e.g. $v_{12}$) are more irrelevant than those displayed. The important result of these equations is that, because of the anisotropic rescaling imposed by the external potential, the third order potentials $w_1$ and $w_2$ are no longer of equal relevancy. $w_1$ becomes relevant at $d = \frac{7}{2}$ whereas $w_2$ and $v_1$ become relevant at $d = \frac{5}{2}$. The relevant potential (for $d < \frac{7}{2}$) $w_1$ can be removed, as in the liquid-gas case, by shifting the order parameter. The upper critical dimension is thus $d_c = 2{\cdot}5$ rather than six (the $B_{20} = 0$ result) or 3·5 (the dimension at which $w_1$ becomes relevant). This means that physical systems in three dimensions will exhibit a mean field transition when $B_{20} \neq 0$. For $d = 2{\cdot}5 - \varepsilon$ the critical point will be in a new universality class with $v_1^* \sim (w_2^*)^2 \sim \varepsilon$.

We now estimate the reduced temperature $t(H)$ at which crossover from symmetric critical to asymmetric mean-field behaviour occurs. Mean field behaviour sets in when the $\chi_a H^2(\nabla_\perp u)^2$ contribution to the free energy exceeds the $K_1(\nabla_\perp^2 u)^2$ contribution to the free energy, i.e. when

$$\chi_a H^2 > K_1 \xi_\perp^{-2}, \tag{10}$$

where $\xi_\perp$ is the perpendicular coherence length. In the vicinity of $H = 0$, it scales as

$$\xi_\perp = t^{-\nu_\perp} f(y(H)/t^\phi) \tag{11}$$

where $y(H) = \chi_z H^2/K_1 \xi_{\perp 0}^{-2}$ is a unitless measure of the strength of the external field and $\xi_{\perp 0} \approx 10\,\text{Å}$ is a bare correlation length. $\phi$ is the crossover exponent associated with $B_{20}$. Rotational invariance leads [8] to $\phi = \beta$ where $\beta$ is the order parameter exponent. To first order in $\varepsilon = 6 - d$, $\beta < 2\nu_\perp$. Thus equation (10) and (11) determine the reduced critical-to-mean-field crossover temperature, $T(H)$ via

$$y(H) \sim t^{2\nu_\perp}(H) f^2(y(H)/t^\phi(H)). \tag{12}$$

If $\phi$ remains less than $2\nu_\perp$ in three dimensions, then

$$t(H) \sim \left(\frac{\chi H^2}{K_1 \xi_{\perp 0}^{-2}}\right)^{1/(2\nu_\perp)}. \tag{13}$$

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If $\phi > 2\nu_\perp$ then the exponent $2\nu_\perp^{-1}$ is replaced by $\phi^{-1}$. If $\chi_a = 10^{-7}$, $H = 10^4$ G, $\xi_{\perp 0} \sim 10_{-7}$ cm, $K_1 \sim 10^{-6}$ dynes, and $\nu_\perp \sim \frac{1}{2}$, this implies

$$t(H) \approx 10^{-7}. \tag{14}$$

The estimate $\nu_\perp \sim \frac{1}{2}$ is crude and may be wrong. In an external electric field, $\chi_a H^2$ is replaced by $\varepsilon_a E^2$. With $\varepsilon_a \sim 10$ cgs and $E \sim 10^2$ stat volt/cm, this yields

$$t(E) \sim 10^{-3} \tag{15}$$

or $T_c - T(E) \sim 10^{-3} \times 400 \sim 0{\cdot}4$ K.

These temperatures should be compared with the Ginzburg temperature $t_G$ for the critical point with $H = 0$ ($E = 0$). If $t(H) > t_G$, ($t(E) > t_G$) the transition will be mean field-like. If $t(H) < t_G$ ($t(E) < t_G$), there will be a crossover from mean field to critical and then back to mean field theory. At the moment, because of our incomplete understanding of the $H = 0$ transition, we are unable to give a believable estimate of $t_G$. Thus, we cannot predict with certainty what the effects of external magnetic or electric fields will have on the $S_A$–$S_{A'}$ transitions. Our educated guess is that there will be critical fluctuations for experimental external magnetic fields but that reasonable electric fields could produce a crossover from critical to mean field behavior or even a purely mean field transition.

Polydomain samples experience strains which we may estimate to be of the order of one lattice spacing $a$ divided by the grain size $L$ independent of temperature: $\langle \nabla_z u \rangle \sim a/L$. These quenched strains create a perturbation whose effect is similar to that of an external electric field. The crossover temperature to mean-field behaviour is again given by equation (13) with $\frac{1}{2}\chi_a H^2$ replaced by $\delta H_{sm}/\delta(\nabla_\perp u)^2 \sim \frac{1}{2} w_2 \langle \nabla_z u \rangle$ where $w_2$ is the unrenormalized potential of equation (5). With $L \sim 10^{-4}$ cm and $w_2 \sim B_2 \sim 10^8$ erg cm$^{-3}$, we find

$$t(\langle \nabla_z u \rangle) \sim 10^{-2} \quad \text{or} \quad T_c - T(\langle \nabla_z u \rangle) \sim 400 \times 10^{-2} = 4\,\text{K}. \tag{16}$$

Thus, quenched strains present in polycrystalline samples may explain the mean-field character of the results of [4].

In conclusion, we have studied the $S_A$–$S_{A'}$ critical point in the presence of an external magnetic (electric) field with positive magnetic (dielectric) anisotropy. We showed that its upper critical dimensions $d_c$ is $\frac{5}{2}$ rather than the zero-field value of 6. Thus, this critical point in physical three-dimensional systems in a field should exhibit mean-field behaviour for temperatures sufficiently close to $T_c$. The reduced temperature at which mean field behaviour sets in are estimated to be $10^{-7}$ for magnetic fields of $10^4$ G, $10^{-3}$ for electric fields of $10^2$ stat volts/cm, and $10^{-2}$ for residual strains of order $3 \times 10^{-3}$.

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